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Strongly Prime Modules In Near-Ring Modules

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Abstract: We deal with primeness in Near-ring modules. In this paper, we introduce the concept of strongly prime module as nonzero module M of a ring R to be strongly prime if $\forall 0 \neq m \in M$, there exists as subset F of R (depending on m) s.t. if $a \in R$ and aFm=0, then a=0 and study several features of this strongly prime ring modules.

1. Introduction: The study of strongly prime modules is done by Handelman and Lawrence Beachy introduced another notion of a strongly prime ring module. Groenewald extended the Handelman- Lawrence definition to near–ring and defined a near-ring R to be right strongly Prime and analogously, a near-ring is defined to be left strongly prime. Furthermore, in ideal P of R is called left strongly prime if R/P is a left strongly prime near-ring. In this section, we generalize these ideas to any R-module M.

2. Preliminaries: In this section, we recall some preliminary definitions and results to be used in the sequel.

2.1 Definition: A nonzero module M of a ring R is said to be strongly prime if for all $0 \neq m \in M$, there exists a finite subset F of R (depending on m) s.t. if $a \in R$ and aFm=0, then a=0

2.2 Definition: A nonzero module M of a ring R is said to be strongly prime (or Beachy-strongly prime) if for each $m' \in M$ and $0 \neq m \in M$, there exists a finite subset F of R s.t. $a \in R$ and aFm=0 implies $am'=0$

2.3 Definition: A near-ring R is said to be right strongly prime if for every $0 \neq a \in R$, there exists a finite subset F of R s.t. if $r \in R$ and aFr=0, then r=0

2.4 Definition: A near-ring R is said to be left strongly prime if for every $0 \neq a \in R$, there exists a finite subset F of R s.t. $r \in R$ and $aFr=0$, then $r=0$.

2.5 Definition: Let M be an R module s.t. $RM \neq 0$, then,

(a) M is said to be (left) strongly prime if for all $0 \neq m \in M$, there exists a finite subset $F = \{ r_1, r_2, \ldots\ldots, r_n \} \subseteq R$ (depending on m) s.t. $a \in R$ and $aFm=0$ implies $aM=0$

(b) An R-ideal P of M is said to be (left) strongly prime if $RM \neq P$ and M/P is a (left) strongly prime module. (i. e. for all $m \in M \backslash P$, there exists a finite subset F of R s.t. $a \in R$ and $aFm=P$ implies $aM=P$).

Hereafter we shall refer to left strongly prime simply as strongly prime. Furthermore, if we refer to a module M as being strongly prime we would mean that it is strongly prime in terms of our definition above. It is quite clear (Proof can be seen in the proposition that follows) that a module M of near-ring R is HL-strongly prime$\Rightarrow$M is Beach-strongly Prime$\Rightarrow$M is strongly prime.

2.6 Definition: An R-module M is said to be cofaithful if there exists a finite subset F of M s.t. $a \in R$ and $aF=0$ implies $a=0$

3.1 Proposition: Let M be an R-moduleof the near-ring R, then the following are equivalent :

(a) M is HL-strongly prime

(b) M is cofaithful and Beachy-strongly prime

(c) M is faithful and strongly prime

Proof:

(a) $\Rightarrow$ (b) : If M is HL-strongly prime, then for each $0 \neq m€M$, there exists a finite $F \subseteq M$ such that $a€R$ and $aFm=0$ implies $a=0$. So for each $m^{/}€M$ it also follows that $am^{/}=0$ and therefore M is Beachy-strongly prime. To show that M is cofaithful, choose $F^{/}=Fm \subseteq M$ and the result follows.

(b) $\Rightarrow$ (c) : Suppose M is cofaithful and Beachy-strongly prime. Since M is cofaithful, it is clearly also faithful and there exists $F^{/}=\{m_1, m_2, \ldots, m_t\} \subseteq M$ such that $r \in R$ and $rF^{/}=0 \Rightarrow r=0$, Let $0 \neq m€M$ then, since M is Beachy-strongly prime, for each $m1 \in F^{/}(1 \leq i \leq t)$ there exists a finite $F_i \subseteq R$ such that $a \subseteq R$ and $aF_i m=0 \Rightarrow am_i=0$. Now let $F=\cup F_i$ Where i= 1, 2, …..t. then $aFm=0 \Rightarrow = \cup F_i =0 \Rightarrow am_i =0$ for all i=1, 2, ….,t.

Thus $aFm=0 \Rightarrow aF^{/}=0 \Rightarrow a=0$. Hence $aM=0$ and M is strongly prime.

(c) $\Rightarrow$ (a) : Since M is strongly prime, for each$0 \neq m€M$, there exists a finite $F \subseteq R$ such that $a \in R$ and $aFm=0$ implies $aM=0$. Since M is faithful, $a=0$ and so M is HL-strongly prime.

3.2 Proposition: If M is a strongly prime R-module, then M is 3-prime.

Proof: Let $a \in R$ and $m \in M$ such that $aRm=0$. Suppose $m \neq 0$. Since M is strongly prime, there exists a finite subset F or R such that $aFm \subseteq aRm=0$ implies that $aM=0$. Hence M is 3-prime.

3.3 Proposition: Let M be a strongly prime R-module, then for every nonzero R-submodule S of M, there exists a finite subset $F=\{s_1, s_2, \ldots, s_n\} \subseteq S$ such that $a \in R$ and $aF=0$ implies $aM=0$

Proof : Let $0 \neq S \leq_R M$ and $0 \neq m \in S$.

Since M is left strongly prime, there exists a finite subset $F=\{r_1,r_2,., r_n\}\subseteq R$ such that $a\in R$ and $aFm=0$ implies that $aM=0$, Let $F_1=F_m==\{r_1m,., r_2m, r_nm\}$. Then $F_1\subseteq S$ since S is an R-submodule of M. Furthermore $aF_1=0\Rightarrow aFm=0$ and hence it follows that $aM=0$.

3.4 Corollary: If R is near-ring with identity then the R-module M is strongly prime if and only if for every if for every nonzero R-submodule S of M, there exists a finite subset $F=\{s_1,s_2,..., s_n\}\subseteq S$ such that $aF=0$ implies $aM=0$

Proof: Let $0\neq m\in M$, Since R has identity $l.m=m\neq 0$. So the proof follows from the previous two propositions.

3.5 Proposition: Let M be a HL-strongly prime R-module. Then for every nonzero R-submodule S of M, there exists a finite subset $F=\{s_1,s_2,..., s_n\}\subseteq S$ such that $a\in R$ and $aF=0$ implies $a=0$.

Proof: follows by a similar argument used in the proof of proposition 3.3

3.6 Proposition: Let M be an R-module such that for every $0\neq m\in M$ there exists an $r\in R$ such that $rm\neq 0$. If for every nonzero R-submodule S of M, there exists a finite subset $F=\{s_1,s_2,..., s_n\}\subseteq S$ such that $a\in R$ and $aF=0$ implies $a=0$, then M is HL-strongly prime.

Proof: Follows by a similar argument used in the proof of above proposition.

3.7 Corollary: If R is a near-ring with identity then the R-module M is HL-strongly prime if for every nonzero R-submodule S of M, there exists a finite subset $F=\{s_1,s_2,..., s_n\}\subseteq S$ such that $aF=0$ implies $a=0$

3.8 Proposition: If R a near-ring with identity and M is an R-module with no nonzero, proper R-submodule then M is Beachy-strongly prime.

Proof: Let $m\in M$ and $0\neq m_1\in M$. Since R has an identity element, we have that $Rm_1=M$. So there exists an $r\in R$ such that $m=rm_1$.If we let $F=\{r\}$ and $aFm_1=0$, then $am=arm_1=0$. Thus M is Beachy-strongly prime.

4. Conclusion: The result in this paper give only the concept of strongly prime module of a ring. Many more information regarding its properties and applications can be expected.

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